

A Model For Late Dark Matter Decay

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The standard cold dark matter cosmological model, while successful in explaining the observed large scale structure of the universe, tends to overpredict structure on small scales. It has been proposed this problem may be alleviated in a class of late-decaying dark matter models, in which the parent dark matter particle decays to an almost degenerate daughter, plus a relativistic final state. We construct explicit particle physics models that realize this goal while obeying observational constraints. To achieve this, we introduce a pair of fermionic dark matter candidates and a new scalar field, which obey either a \mathbb{Z}_4 or a $U(1)$ symmetry. Through the spontaneous breaking of these symmetries, and coupling of the new fields to standard model particles, we demonstrate that the desired decay process may be obtained. We also discuss the dark matter production processes in these models.

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I. INTRODUCTION

There is an abundance of evidence to indicate the existence of dark matter (DM), including its necessary contribution to both galactic stability and structure formation in the early Universe [1–3]. The standard Λ CDM cosmological model, in which cold dark matter (CDM) makes up 22% of the universal energy budget, provides an excellent description of our Universe. However, little is known about the particle properties of dark matter. In addition, some problems with CDM are encountered at small scales.

A popular class of CDM candidates is weakly interacting massive particles (WIMPs). In WIMP models the DM couples weakly to standard model (SM) particles, which allows for scattering/annihilation processes. These serve to keep the dark sector in thermal equilibrium with the visible sector in the early Universe and can, with an appropriate choice of coupling, cause the DM to freeze out with the correct relic density. Interaction with the SM is similarly appealing from a detection standpoint, potentially providing both direct [4–6] and indirect [7–9] signatures of a given model. Nonobservation of these signatures allows for constraints to be placed on parameters such as particle masses or coupling constants.

Though the DM mass is unknown, some information can be inferred from observations of large scale structure. For cold dark matter, structure forms hierarchically, with the earliest structures formed on short length scales, which can then merge to form larger structures. This is to be contrasted with hot dark matter in which the largest superclusters form first. Numerical simulation has shown the CDM scenario to fit observations well [10], while hot dark matter is strongly disfavored.

The CDM model is not-problem free, however, as it tends to overproduce small scale power [10–24]. Simulations predict cusps in the DM density at the centers of galactic halos in conflict with observation. CDM also over-predicts the number of dwarf galaxies orbiting a Milky Way-sized galaxy by about a factor of 10. Al-

though simulations do not include visible matter, the gravitational potential wells they predict would promote a level of star formation not observed. Though these issues may be partially alleviated by tidal disruption and other effects, the small scale power problems of Λ CDM are still poorly understood (see e.g. [25, 26] for recent work). Such issues have led many to consider a “warm” DM candidate, with a mass of keV scale, intermediate between hot and cold dark matter. In this work, as in [27–34], we will consider an alternative hypothesis in which the usual assumption of a single DM candidate is challenged.

We consider a scenario with two WIMP candidates, in which one species is unstable to decay into the other. If the mass splitting between the two WIMPs is sufficiently small, the decay process will leave the overall halo mass unaffected, while giving its constituent DM particles a small velocity kick. Such velocity kicks heat the dark matter halos and cause them to expand, softening the central cusps and disrupting small halos [35–39]. Such models are appealing, as they can alleviate the small scale structure problems, while retaining the attractive features of cold dark matter.

We shall assume the DM decays predominantly via the channel

$$\chi^* \rightarrow \chi + l, \quad (1)$$

where χ^* and χ denote the heavier and lighter candidate, respectively, and l is some relativistic final state. The mass splitting between χ^* and χ is given by

$$\Delta m = m_{\chi^*} \epsilon, \quad (2)$$

where $\epsilon \ll 1$. Abdelqader and Melia [35] have shown the dwarf halo problem can be solved for $\epsilon \simeq (5 - 7) \times 10^{-5}$ and a decay lifetime of $(1 - 30)$ Gyr. The work of Peter, Moody, and Kamionkowski [36] has demonstrated that galaxy cusps can be alleviated for a wider range of ϵ and τ_{χ^*} , with the most favored lifetimes in the range $(0.1 - 100)$ Gyr. Subsequent work by Peter and Benson [40] has used properties of galactic subhalos to further constrain the allowed values of ϵ , preferring lower

values to those favored in [35]. Dark matter decays may be further constrained from analysis of their effect on weak lensing of distant galaxies as in [41]. However, at present such analyses have only placed limits on models with much larger values of ϵ than those considered in this work.

An interesting possibility, from an observational standpoint, is a decay mode in which the relativistic final state, l , consists of SM particles. This allows the possibility of verifying the model, via the detection of particles produced by decay in our own Galaxy, or of a diffuse flux from decays in halos throughout the Universe. Current astrophysical observations place constraints on the allowed parameters, via comparison of the decay fluxes with relevant astrophysical backgrounds. Reference [42] placed stringent constraints on the decay parameters for the case in which l is a photon, while Ref. [43] derived somewhat weaker constraints for the cases in which $l = \bar{\nu}\nu$ or e^\pm . For $l = \gamma$ or e^\pm the lifetime is restricted to be below about 1 Gyr, while a much larger range of lifetimes is permitted for $l = \bar{\nu}\nu$.

The aim of this work is to construct a particle physics model which can realize the decaying dark matter scenario. We shall use the criterion specified by Abdelqader and Melia [35] [namely, $\epsilon \simeq (5-7) \times 10^{-5}$ and $\tau \sim (1-30)$ Gyr] as a reference point for these models, but given the constraints of Ref. [40], we will choose the more restrictive value of $\epsilon \sim 10^{-5}$ and $\tau \simeq (1-10)$ Gyr. In Sec. II we introduce and discuss two possible models for decaying dark matter, and outline the DM production mechanism. Section III focuses on constraints on the models and the available regions of parameter space. We conclude in Sec. IV.

II. A DARK MATTER DECAY MODEL

In order to construct a model which can achieve this decay scenario there are certain criteria that need to be satisfied. The first and most important of these is the need for two candidates with nearly degenerate masses. Second, we need either decay of the parent DM particle to light SM final states, or to some new light degree of freedom. Third, the process needs to occur on times scales relevant for the disruption of structure formation, and last, we need some viable DM production mechanism. WIMP-like scenarios are particularly interesting on this front, as WIMPs are populated as thermal relics and naturally freeze out in the early Universe with the correct relic density.

Two scenarios will be considered. In the first we implement a variation of the “exciting dark matter” model conceived by Finkbeiner and Weiner [27], which involves the addition of a dark sector containing a Dirac fermion and a real scalar field to the standard model. The introduced fields obey a discrete \mathbb{Z}_4 symmetry, the breaking of which leads to a nondegeneracy of the masses of the fermion’s two Weyl components, and an instability of the

heavier to decay into the lighter. The scenario considered in this work differs from [27] in the values of the model parameters chosen; in short, we consider longer decay lifetimes.

As a second example, we consider a generalization of the scenario in [27], in which we replace the \mathbb{Z}_4 symmetry with a global $U(1)$ symmetry, requiring the introduced scalar field to be complex. The breaking of this $U(1)$ will produce a pseudo-Nambu-Goldstone boson, which will serve as our light final state for the dominant decay channel. We show that production through interaction with the SM is impossible in the $U(1)$ model. The second scenario is one example among any number of generalizations and extensions to the simple \mathbb{Z}_4 model; it is simply an illustration that decaying DM can be realized in a particle model irrespective of the strength of coupling to the SM.

In Sec. II A we shall explore a model in which SM final states are produced in the dominant decay channel. In II B we consider the possibility of completely non-SM final states. In II C we discuss production, and in II D consider the possibility of χ^* depopulation.

A. SM Final States (\mathbb{Z}_4)

When searching for a light final state for the process in Eq.(1) the obvious place to look is the SM, as the existence of particles with masses substantially below the CDM mass scale (GeV) is assured. To couple the DM to the SM we adopt the model put forward in [27]. Although this was originally intended as a mechanism for explaining the observed INTEGRAL/SPI positron excess [9], with a different choice of parameters the model can serve our astrophysical aims quite well. We begin with the introduction of a Dirac fermion comprised of the two Weyl spinors χ_{1L} and χ_{2R} , which couple to a real singlet scalar ϕ . The mass eigenstates for the χ_1 and χ_2 fields (which we will call χ and χ^* , respectively) are the DM in this model.

We impose a discrete \mathbb{Z}_4 symmetry under which the fields transform as

$$\begin{aligned}\chi_{1,2} &\rightarrow i\chi_{1,2}, \\ \phi &\rightarrow -\phi,\end{aligned}\tag{3}$$

but remain singlets under the symmetries of the SM. This allows for the following Lagrangian:

$$\begin{aligned}\mathcal{L} = & \frac{1}{2}\partial_\mu\phi\partial^\mu\phi + \sum_i^2 \chi_i^\dagger\sigma_\mu\partial^\mu\chi_i - m_D\overline{\chi}_{1L}\chi_{2R} \\ & - \lambda_1\phi\overline{\chi}_{1L}(\chi_1)_R^c - \lambda_2\phi\overline{\chi}_{2R}(\chi_2)_L^c - V(\phi, H, H^\dagger) + h.c.\end{aligned}\tag{4}$$

At this stage the two mass eigenstates both have mass m_D . To lift this degeneracy we need to break the \mathbb{Z}_4 symmetry. We break the symmetry spontaneously down to \mathbb{Z}_2 by allowing ϕ to obtain a vacuum expectation value.

The Higgs potential is given by

$$V(\phi, H, H^\dagger) = \frac{\lambda_\phi}{4} \phi^4 - \frac{\mu_\phi^2}{2} \phi^2 + \frac{\lambda_h}{4} (H^\dagger H)^2 - \frac{\mu_h^2}{2} H^\dagger H + \frac{\alpha}{2} \phi^2 (H^\dagger H), \quad (5)$$

where H is the SM Higgs doublet. The last term in Eq.(5) is included as it is allowed by all the symmetries of the theory. Minimizing the above potential with respect to both ϕ and H we obtain the conditions

$$2\lambda_\phi \langle \phi \rangle^2 - 2\mu_\phi^2 + \alpha \langle h \rangle^2 = 0, \\ \lambda_h \langle h \rangle^2 - 2\mu_h^2 + 2\alpha \langle \phi \rangle^2 = 0. \quad (6)$$

It can now clearly be seen that $\langle \phi \rangle \neq 0$. As in [27], the spontaneous breaking of the discrete \mathbb{Z}_4 symmetry will lead to the formation of domain walls, which may be disfavored by observation. We can remove this potentially troubling phenomenon by introducing the explicit breaking term $\mu\phi^3$ to our Higgs potential, where μ is small for reasons of technical naturalness.

Perturbing Eq.(4) about the vacuum there arise Majorana masses $\lambda_1 \langle \phi \rangle$ and $\lambda_2 \langle \phi \rangle$ for χ_1 and χ_2 , respectively. The Lagrangian therefore contains the mass matrix

$$\mathcal{L} \supset - \begin{pmatrix} \overline{\chi_{1L}} & (\overline{\chi_{2R}})^c \end{pmatrix} \begin{pmatrix} \lambda_1 \langle \phi \rangle & \frac{m_D}{2} \\ \frac{m_D}{2} & \lambda_2^* \langle \phi \rangle \end{pmatrix} \begin{pmatrix} (\chi_{1L})^c \\ \chi_{2R} \end{pmatrix} \quad (7)$$

which we then diagonalize to obtain the Majorana mass eigenstates χ and χ^* :

$$\chi \simeq \frac{1}{\sqrt{2}} [(\chi_1 + \chi_2)^c - (\chi_1 + \chi_2)], \\ \chi^* \simeq \frac{1}{\sqrt{2}} [(\chi_1 + \chi_2)^c + (\chi_1 + \chi_2)], \quad (8)$$

whose masses we find to be

$$m_{\chi^*, \chi} = \frac{1}{2} \sqrt{m_D^2 + 4\lambda_\pm^2 \langle \phi \rangle^2} \pm \lambda_\pm \langle \phi \rangle, \quad (9)$$

where $\lambda_\pm \equiv \frac{1}{2}(\lambda_1 \pm \lambda_2^*)$. We want the mass splittings to be small, so we choose $m_D \gg \lambda_\pm \langle \phi \rangle$, making $m_{\chi^*, \chi} \simeq \frac{m_D}{2} \pm \lambda_\pm \langle \phi \rangle$, and thus $m_{\chi^*} \epsilon = 2\lambda_+ \langle \phi \rangle$. Typically in this model we shall consider masses in the range $m_{\chi, \chi^*} \sim (50-800)$ GeV, a breaking scale of $\langle \phi \rangle \sim (3-20)$ MeV, and coupling strength of $\lambda_\pm \sim 10^{-1}$ [implying $\Delta m \sim (0.4-8)$ MeV for $\epsilon \simeq (0.7-1) \times 10^{-5}$]. For a detailed discussion of the parameter space, see Sec. III.

In the basis of the mass eigenstates, the Lagrangian contains the following interaction terms, which mediate both decay and scattering or annihilation processes:

$$\mathcal{L} \supset \lambda_+ \phi \overline{\chi} \chi - \lambda_+ \phi \overline{\chi^*} \chi^* - \lambda_- \phi \overline{\chi} \gamma_5 \chi^* - m_\chi \overline{\chi} \chi - m_{\chi^*} \overline{\chi^*} \chi^* + h.c. \quad (10)$$

It should be noted that interaction terms coupling like mass eigenstates are scalar, while off-diagonal coupling is pseudoscalar. As will be seen this has a substantial effect on the DM decay rate.

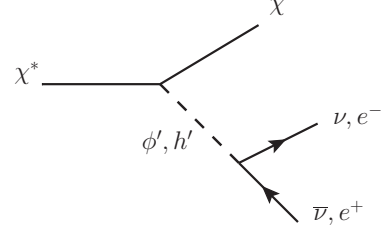


FIG. 1. Primary DM decay channel for the \mathbb{Z}_4 model

Mixing of the SM sector with the dark sector (χ, χ^*, ϕ) occurs through the last term in Eq.(5). Expanding the Higgs potential about the vacua of both ϕ and H produces off-diagonal mass terms for both fields. Expressing the potential in terms of the mass eigenstates ϕ' and h' , we find the following mixing of states:

$$\phi \simeq \cos \theta \phi' - \sin \theta h' \simeq \phi' - \theta h', \quad (11) \\ h \simeq \cos \theta h' + \sin \theta \phi' \simeq h' + \theta \phi',$$

where h is the SM Higgs boson, and λ_h is its self-coupling. To first order in α

$$\theta \simeq \frac{\alpha \langle \phi \rangle}{\lambda_h \langle h \rangle}. \quad (12)$$

and we find masses of ϕ' and h' to be

$$m_\phi^2 \simeq 2\lambda_\phi \langle \phi \rangle^2, \\ m_h^2 \simeq \frac{1}{2} \lambda_h \langle h \rangle^2, \quad (13)$$

(in the limit $m_\phi \ll m_h$) with that of ϕ' being $\sim (2-20)$ MeV. In this work we adopt a SM Higgs mass of $m_h \sim 130$ GeV. Through ϕ -Higgs mixing χ and χ^* can couple to h' and by extension the SM. In particular, it allows the possibility of decay into SM final states via processes such as that shown in Fig.1.

This process has a decay rate given by

$$\Gamma \simeq \frac{\lambda_-^2 y_l^2 \theta^2}{2800 \pi^3 m_\phi^4} m_{\chi^*}^5 \epsilon^7, \quad (14)$$

where y_l is the Yukawa coupling for the dominant SM final state, and we assume $\Delta m \gg m_l$, $\lambda_+ \simeq \lambda_-$, and that the light final states are Dirac. Just as we choose our splitting to be small, we can choose the region of parameter space in which the lifetime is sufficiently large to disturb structure formation.

The above decay rate contains several elements which naturally lead to suppression and thus a long lifetime. First, the decay rate is subject to phase-space suppression, as there are 3 bodies in the final state. Second, it depends on the Yukawa coupling y_l , which, given we are only interested in decay into light leptons ($e^+ e^-$, $\bar{\nu} \nu$), will be a small number. It is also dependent on the Higgs mixing angle θ , which in turn varies depending on the

strength of the coupling α . Although there is some freedom of choice with respect to the value of α , we typically take $\alpha \sim 10^{-5}$, which results in a mixing of $\theta \sim 10^{-9}$ (see Sec. II C for details). Lastly, pseudo-scalar coupling between χ and χ^* means the decay rate contains a factor ϵ^7 (as opposed to ϵ^5 for scalar coupling). The conjunction of these factors means that the DM lifetime can be long without λ_{\pm} being too small, typically $\lambda_{\pm} \sim 10^{-1}$.

B. Dark Decays [$U(1)$]

An advantage of SM final states is their detectability. While directly observable consequences are a desirable model building goal, the nonobservation of the signatures of a model can lead to constraints, as will be seen in the next section. Were observational constraints to strengthen, there is the possibility that a nonobservation of the final states in the above model may rule it out. Should this occur the viability of decaying DM in general would rely on the primary decay channel being independent of the SM. In this section we will present a model which can realize this.

One way to naturally produce a decay channel with a light final state is to upgrade the discrete \mathbb{Z}_4 symmetry to a global $U(1)$ symmetry. Spontaneously breaking this will produce a Nambu-Goldstone boson (NGB) in the theory, which will couple to the DM. The Lagrangian in this scenario is similar to Eq.(4) except that now $\phi = (1/\sqrt{2})(\phi_1 + i\phi_2)$, and $\chi_{1,2}$ and ϕ now transform under the $U(1)$ symmetry as

$$\begin{aligned}\chi_{1,2} &\rightarrow \chi_{1,2} e^{i\theta_\chi}, \\ \phi &\rightarrow \phi e^{-2i\theta_\chi},\end{aligned}\quad (15)$$

where θ_χ is some arbitrary phase. The Higgs potential will have a similar form to Eq.(5) only now $\phi \neq \phi^\dagger$, and ϕ^2 terms become $\phi^\dagger\phi$. As with the discrete case, we end up with the mass matrix in Eq.(7) and subsequent mass eigenstates χ and χ^* ; only now they couple to both the mass eigenstates ϕ'_1 defined by

$$\begin{aligned}\phi_1 &\simeq \cos\theta_1 \phi'_1 - \sin\theta_1 h' \simeq \phi'_1 - \frac{\alpha\langle h \rangle}{\lambda_\phi\langle\phi\rangle} h', \\ h &\simeq \cos\theta_1 h' + \sin\theta_1 \phi'_1 \simeq h' + \frac{\alpha\langle h \rangle}{\lambda_\phi\langle\phi\rangle} \phi'_1,\end{aligned}\quad (16)$$

($m_\phi \gg m_h$ as will be seen) and the NGB ϕ_2 , i.e.

$$\mathcal{L} \supset -\frac{\lambda_-}{\sqrt{2}}\phi'_1\bar{\chi}\gamma_5\chi^* - \frac{i\lambda_+}{\sqrt{2}}\phi_2\bar{\chi}\chi^* + h.c. \quad (17)$$

It should be noted that the above coupling to the NGB is scalar, which results from the fact that ϕ_2 is the imaginary component of ϕ . This means that decays into the NGB contain less ϵ suppression (one power of ϵ) than they would were the coupling pseudoscalar (ϵ^3).

Coupling to the NGB implies the existence of long range DM-DM interactions, which can potentially affect structure formation. To avoid the issues involved

with this, we will introduce a small soft breaking term, $\frac{\mu^2}{2}(\phi^2 + \phi^{\dagger 2})$ to the Higgs potential to explicitly break the continuous $U(1)$ symmetry down to the discrete \mathbb{Z}_4 . This gives the NGB an $\mathcal{O}(\mu)$ mass, which is naturally small.

The off-diagonal interaction term in Eq.(17) leads to the decay channel $\chi^* \rightarrow \chi + \phi_2$, which has the decay rate

$$\Gamma \simeq \frac{\lambda_+^2}{4\pi} m_{\chi^*} \epsilon. \quad (18)$$

For the case where the primary decay of the DM was into SM final states, the reason for a long lifetime was the weak mixing with the SM and the suppression from the high power of ϵ . Should, however, the decay referred to in Eq.(18) be the primary channel, there is no such suppression, and we are forced to impose the additional approximate symmetry $\lambda_1 \simeq -\lambda_2^*$ to make $\lambda_+ \sim \mathcal{O}(10^{-18})$, hence achieving $\tau \sim \mathcal{O}(\text{Gyr})$. As $\Delta m = 2\lambda_+\langle\phi\rangle$, small λ_+ implies a high breaking scale for reasonable values of the DM mass, roughly $\langle\phi\rangle \sim 10^{14}$ GeV. As will be discussed in II C 2, $\lambda_\phi \sim 1$ making $m_\phi \sim \langle\phi\rangle$ in this model.

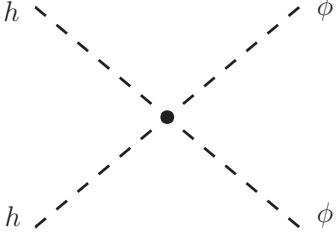
This high scale has the potential to cause problems. Recall the minimization conditions in Eq.(6). In order to reproduce the correct breaking scale for the SM Higgs, either α needs to be small enough such that $\alpha\langle\phi\rangle^2$ is negligible with respect to μ_h^2 , or we have a finely tuned scenario resembling the hierarchy problem of the SM, in which $\alpha\langle\phi\rangle^2$ and μ_h^2 are of similar order. The former, though the more natural of the two cases, precludes production via mixing with the SM, so we will entertain the latter for the time being.

C. Production

Both scenarios presented have all the required elements to disrupt structure formation in the desired fashion. All that is needed now is a production mechanism for the dark matter candidate in each scenario. As mentioned previously, one of the appealing properties of a WIMP is attainment of the correct relic abundance through thermal freeze-out from the bath in the early Universe.

1. \mathbb{Z}_4 Case

In the model presented in II A, χ couples to the SM through the Yukawa sector. It therefore follows that it is through these channels that it will maintain equilibrium with the SM prior to freeze-out. Production differs from the standard WIMP scenario in that it is a two-phase process. The ϕ is populated via interactions with the SM in the Higgs sector, while χ and χ^* are produced through their coupling to ϕ . At some temperature below the DM mass the ϕ - χ annihilation rate will drop below the expansion rate, and χ and χ^* will freeze out with respect to ϕ , fixing the comoving DM abundance to the

FIG. 2. Dominant ϕ production mechanism for $T > m_h$.

standard value. We will now chronologically step through the processes leading to DM freeze-out.

The requirement that ϕ be in chemical equilibrium with the SM well before χ - ϕ freeze-out places a constraint on the allowed values of the coupling α . Well above the electroweak scale, the dominant process keeping ϕ in chemical equilibrium with the SM will be $hh \rightarrow \phi\phi$ (Fig.2), which at temperatures well above the Higgs mass has the annihilation rate

$$\Gamma(hh \rightarrow \phi\phi) \simeq \frac{\alpha^2 T}{256\pi^3}. \quad (19)$$

This process will remain in equilibrium until $T < m_h$, and h production becomes Boltzmann suppressed, causing this ϕ production channel to become unavailable. For the DM masses of interest in this model (of order or below m_h), we require $\alpha > 10^{-6}$ to ensure that ϕ is in equilibrium at some point prior to ϕ - χ freeze-out.

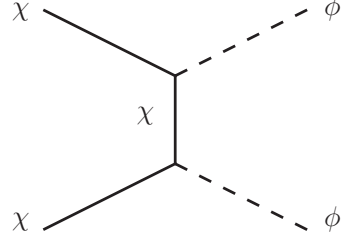
Also contributing to ϕ production is the h -mediated processes $\bar{f}f \rightarrow \phi\phi$, which have the annihilation rate of

$$\Gamma(f\bar{f} \rightarrow \phi\phi) \simeq \frac{\alpha^2 y_f^2 \langle h \rangle^2 T^3}{16\pi^3 m_h^4}. \quad (20)$$

up to a color factor for processes involving quarks. The temperatures at which these processes freeze out depend on both the Higgs mixing α and SM fermion Yukawa y_f . For the values of α considered in this paper ($\alpha \sim 10^{-5}$) the process which remains in equilibrium longest is that involving b quarks. This freezes out around the time at which the annihilation in Fig.2 turns off. Thus the temperature at which ϕ freezes out with respect to the SM can be calculated to be $T_f^{\phi-SM} \sim 20$ GeV. This occurs when ϕ is still relativistic.

After ϕ -SM freeze out, the temperature of the ϕ - χ system will continue to track that of the background.¹ The

¹ Up to a factor $(g'_*/g_*)^{1/3}$, where g_* and g'_* are measures of the number of freedom in the bath at $T_f^{\phi-SM}$ and $T_f^{\phi-\chi}$ respectively. We will assume this ratio to be ~ 1 , and $\sqrt{g_*} \simeq 10.8$, i.e. that all degrees of freedom are in equilibrium. While depending on the time of DM freeze out this might not be strictly true, the effect on the results will be negligible. It is therefore irrelevant exactly when ϕ freezes out with respect to the SM, as long as it has been in equilibrium at some point prior to ϕ - χ freeze out.

FIG. 3. DM produced through ϕ annihilation.

DM will be kept in chemical equilibrium with ϕ through the scattering in Fig.3, which in the nonrelativistic limit has a cross section of

$$\sigma v_{rel} \simeq \frac{|\lambda_+|^4}{\pi m_\chi^2}. \quad (21)$$

This process will freeze out once the temperature of the ϕ - χ system falls below m_χ and the number density of χ becomes Boltzmann suppressed.

To determine the DM relic abundance we use the well established result [44]

$$\Omega_\chi h^2 = 1.07 \times 10^9 \frac{x_{DM-f} \sqrt{g_*} GeV^{-1}}{g_{*s} m_{Pl} \langle \sigma v \rangle}, \quad (22)$$

where

$$x_{DM-f} = m_\chi / T_f^{\phi-\chi} = \ln[0.038(g/\sqrt{g_*})m_{Pl}m_\chi \langle \sigma v \rangle] - \frac{1}{2} \ln[\ln[0.038(g/\sqrt{g_*})m_{Pl}m_\chi \langle \sigma v \rangle]], \quad (23)$$

and $T_f^{\phi-\chi}$ is the temperature at which ϕ and χ drop out of chemical equilibrium. We find that typically $x_{DM-f} \sim 20$. The requirement that we produce the observed relic density of $\Omega_\chi \simeq 0.22$, places constraint on the free parameters λ_+ and m_χ should the DM be a thermal relic. See Sec. III for a full treatment of the parameter space. Given that $m_\chi \simeq m_{\chi^*}$ (and $\Delta m \ll T_f^{\phi-\chi}$) χ and χ^* will be produced in equal abundance.

After ϕ - χ freeze-out, the relativistic ϕ will remain with fixed abundance until the spontaneous breaking of the \mathbb{Z}_4 symmetry (at MeV scale). After symmetry breaking they become unstable to decay into photons via the loop order process depicted in Fig. 4 [45, 46]. This process has a rate of

$$\Gamma(\phi \rightarrow \gamma\gamma) \simeq \frac{G_F \alpha_{EM}^2 \theta^2 M_W^4}{2\sqrt{2}\pi^3 m_\phi} \quad (24)$$

$$\simeq 4.5 \times 10^4 s^{-1} \left(\frac{\theta}{10^{-9}} \right)^2 \left(\frac{10 MeV}{m_\phi} \right),$$

which is large compared to the expansion rate, and ϕ rapidly depopulates.

In our calculation of the process depicted at tree level in Fig. 3, we have omitted the contribution from ladder diagrams involving ϕ exchange in the initial state.

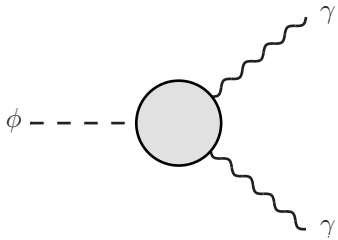


FIG. 4. 1-loop order decay $\phi \rightarrow \gamma\gamma$, through h - ϕ mixing. Includes contribution from loops involving W^\pm , unphysical charged Higgs components h^\pm , and Fadeev-Popov ghosts.

This approximation is valid at high energies, but begins to break down near freeze out, when the DM is in the moderate-nonrelativistic regime.

At low velocity the Yukawa potential (resulting from ϕ exchange) from one initial state χ can significantly distort the wave-function of the other from that of a free particle. This leads to an enhancement of the velocity averaged cross section in an effect known as Sommerfeld enhancement [28, 47–49]. This effect can be taken into account by multiplying the relevant cross section by a velocity dependent Sommerfeld factor S . To calculate the enhancement to the process in Fig. 3 we follow the method of [47, 48], but find that in the relevant region of parameter space S is close to 1 and the enhancement negligible. The enhancement generally becomes more important for larger values of Δm .

2. $U(1)$ Case

Production in the second model presented is slightly more difficult. An unfortunate consequence of a small Yukawa coupling is a weakening of the annihilation cross section (Fig. 3). This suppression ensures that the process in Fig. 3 is never in equilibrium, making thermal production of the DM impossible. This leads us to consider a nonthermal production mechanism, in which χ and χ^* are produced out of equilibrium through their weak mixing with the bath. Another possibility is production through direct coupling of the DM to the inflaton. While this is clean in that it is independent of SM processes, it requires fine tuning to attain the correct relic abundance. For the time being we will entertain the former possibility.

The dominant channel through which production can occur is through the SM Higgs annihilation pictured in Fig. 5. As $\langle\phi\rangle$ is large in this model, we expect this process to be strongest above the electroweak breaking scale. At these high temperatures finite temperature effects come into the Higgs potential at loop order [50–52]. This has the effect of giving the scalar components of the SM Higgs doublet temperature-dependent masses of the

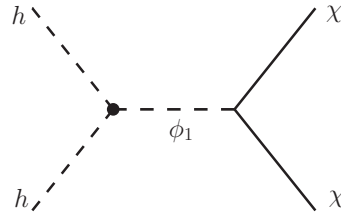


FIG. 5. Dominant DM production mechanism in the $U(1)$ model.

form

$$m_h^2 \simeq \frac{\lambda_h T^2}{24}. \quad (25)$$

The process in Fig. 5 goes to a maximum near the ϕ_1' resonance, in which region $m_{\phi_1}^2 \simeq 4m_h^2$. Granted $\alpha^2 \ll \lambda_\phi$, and following the analysis of [53], the velocity averaged cross section can in this region be well approximated by

$$\langle\sigma v\rangle \simeq \frac{\lambda_+^2 T m_\phi^3}{\pi^3 (n_h^0)^2} K_1\left(\frac{m_\phi}{T}\right) \times \frac{\frac{\alpha^2}{\lambda_\phi} \sqrt{m_\phi^2 - 4m_h^2}}{\left[\left(\frac{\alpha^2}{\lambda_\phi} \sqrt{m_\phi^2 - 4m_h^2} + \lambda_\phi m_\phi\right) \coth\left(\frac{m_\phi}{4T}\right) + 128\lambda_+^2 m_\phi\right]}. \quad (26)$$

To avoid Boltzmann suppression in Eq. (26) we will take λ_ϕ to be small for now ($\lambda_\phi \sim 10^{-17}$), which implies that m_{ϕ_1} is far below the $U(1)$ breaking scale ($m_{\phi_1} \sim 100$ TeV). In order to calculate the abundance at a particular temperature, we must solve the comoving Boltzmann equation, which can be expressed in the form

$$\frac{dn_\chi(T)}{dT} - \frac{3}{T} n_\chi(T) = -\frac{(n_h^0)^2}{HT} \langle\sigma v\rangle, \quad (27)$$

and has the solution

$$n_\chi(T) = T^3 \int_T^{T_{n_\chi=0}} \frac{(n_h^0)^2 \langle\sigma v\rangle}{HT'} dT', \quad (28)$$

where $T_{n_\chi=0} \simeq \sqrt{6/\lambda_h}$ is defined by the temperature at which $m_{\phi_1}^2 \simeq 4m_h^2$ and will be taken to be when significant production starts.

For the representative region of parameter space $\lambda_\phi \sim 10^{-17}$, $\alpha \sim 10^{-15}$, $\lambda_+ \sim 10^{-18}$, and $\tau_{\chi^*} \sim 1$ Gyr, the comoving number density can be calculated to be $\mathcal{O}(10^{-27})$, roughly 17 orders of magnitude below the required value at that temperature. These values for the parameters in the model were chosen as they were shown to maximize production. As this channel is expected to be the strongest available it is therefore clear that production of the DM via mixing with the SM in such a model is impossible. The implication of neither a SM final state nor SM related production is the independence of the dark sector from the visible. This gives us complete freedom in the choice of dimensionless parameters α and λ_ϕ but

precludes entirely the possibility of direct verification of the model. We can now choose $\lambda_\phi \sim 1$ and α to be very small to avoid issues of fine-tuning.

Independence of the dark sector from the SM implies the necessity for some novel DM production mechanism. As mentioned earlier this can be realized through a direct coupling of the DM to the inflaton, but as stated such a mechanism is problematic as it is difficult to obtain a relic density of order that of the SM without fine-tuning of the DM-inflaton coupling.

D. Depopulation of the Excited State

In the \mathbb{Z}_4 model, as the temperature of the ϕ - χ system drops well below the DM mass, χ and χ^* will have chemically frozen out fixing the relic abundance. The s-channel equivalent to Fig. 3 will, however, maintain kinetic equilibrium in the ϕ - χ system to temperatures down as low as m_ϕ [47, 54]. Both χ and χ^* will be kept in equilibrium with each other by way of a process like that in Fig. 6, causing both to track closely the temperature of the background. However, as the average kinetic energy drops below Δm the process $\chi\chi \rightarrow \chi^*\chi$ is no longer kinematically viable, and the up-scattering rate becomes Boltzmann suppressed [54]. The result is a rapid depopulation of χ^* , and an absence of the heavy state so important for disturbance of structure formation. This issue can be averted should the scattering rate for Fig. 6 be small enough such that the process freezes out sufficiently early, i.e for $T \gg \Delta m$. Should this be the case, both the forward and back scattering processes will cease well before depopulation becomes an issue.

The cross section for this process (at tree level) can be calculated to be

$$\sigma v_{rel} \simeq \frac{3|\lambda_- \lambda_+|^2}{\pi m_{\chi^*}^2} \frac{1}{v_{rel}} \log \left[\frac{32}{v_{rel}^2} \right], \quad (29)$$

in the limit $m_\phi^2 \ll m_{\chi^*} \Delta m$, which is justified in the region of parameter space considered (see Sec. III). In the moderate to nonrelativistic regime, the scattering rate for process $\chi^*\chi \rightarrow \chi\chi$ is given by

$$\Gamma \simeq (n_\chi) \frac{x_{sc-f}^{3/2}}{2\sqrt{\pi}} \int_0^1 (\sigma v_{rel}) S v_{rel}^2 e^{-x_{sc-f} v_{rel}^2/4} dv_{rel} \quad (30)$$

where $x_{sc-f} = m_\chi/T_f^{\chi\chi^*}$, and $T_f^{\chi\chi^*}$ is defined as the temperature at which the process in Fig. 6 freezes out.

After the process in Fig. 3 freezes out, the comoving DM number density n_χ/T^3 is fixed, and is given by

$$n_\chi/T^3 \simeq \frac{g_{*s}}{m_\chi} 3.76 \times 10^{-11} \text{ GeV}.$$

where T is the temperature of the bath. We can now choose parameters such that the process in Fig. 6 freezes out around the same time as that of Fig. 3, in which case Eq.(21) and Eq.(29) are of a similar order. In the relevant region of parameter space, this is generally the

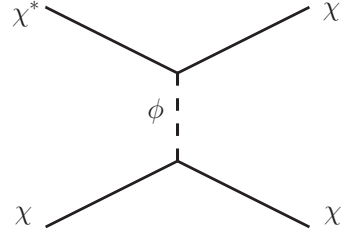


FIG. 6. Process by which χ and χ^* maintain chemical equilibrium.

case, with $x_{sc-f} \sim 1$. Interestingly, this is before ϕ - χ freeze-out, meaning χ and χ^* are both in equilibrium with ϕ but not each other.

In the above, we have considered only the depopulation of χ^* in the early Universe. It is also important that χ^* not be depopulated via scattering in the late Universe, when Sommerfeld effects are significant. In fact, additional constraints on $\chi\chi$ and $\chi\chi^*$ scattering arise from the requirement that self-scattering of DM does not significantly perturb galactic halo shapes [55]. These requirements will be taken into account in Sec. III.

In the $U(1)$ model there are no such depopulation issues, as λ_+ is very small and the DM is never in equilibrium in the first place.

III. CONSTRAINTS ON \mathbb{Z}_4 MODEL

Up to this point there has been minimal discussion of the choice of values for the many free parameters in our model. In order to do so clearly it is important to understand exactly what constraints are present. There are initially 7 independent free parameters, those related to the fermions χ and χ^* , namely, m_{χ^*} and λ_\pm , and those belonging to the Higgs sector: λ_ϕ , α , μ_ϕ , and μ . Recall also that we can express the mass splitting in terms of these parameters, that is, $\Delta m = 2\lambda_+ \langle \phi \rangle$ [$\langle \phi \rangle$ depends on Higgs potential parameters from Eq.(6)]. Thus when we parametrize Δm in terms of ϵ ($\Delta m = m_{\chi^*} \epsilon$) and fix its value to $\epsilon = 10^{-5}$, we place a constraining relationship between λ_+ , m_{χ^*} , and $\langle \phi \rangle$. As a second constraint we will impose $\lambda_+ \sim \lambda_-$, as they will only differ greatly in the finely tuned scenario where $\lambda_1 \simeq \pm \lambda_2$ to high precision. Last, we must satisfy the condition in Eq.(22), to ensure correct relic abundance. These three constraints reduce the number of free independent parameters to 4, which can be taken to be $\langle \phi \rangle$, λ_ϕ , α , and μ . We can now express allowed values of Δm as a function of breaking scale $\langle \phi \rangle$ for chosen values of α and λ_ϕ . We must choose α appropriately such that ϕ goes into equilibrium with the bath before the temperature of DM freeze-out. The allowed values of Δm for the appropriate DM lifetimes are plotted on the left hand side of Figs. 7- 8, while the corresponding values of m_{χ^*} (for $\epsilon = 10^{-5}$ and $\epsilon = 0.7 \times 10^{-5}$

for Figs. 7 and 8, respectively) are plotted on the right.

The presence of readily detectable charged particles in the final state increases the possibility of both direct and indirect detection. Indeed heavy constraints can be placed on the parameter space based on nonobservation of the consequences of such a final state. In [42, 43] detailed analyses of the photon, positron and neutrino backgrounds were performed with decaying DM models in mind, and constraints placed on the relevant parameters τ_{DM} and Δm . We have translated the constraints on decay to e^+e^- to the parameter space relevant to this model, resulting in the exclusion region in Fig. 7.

Which leptons will be produced predominantly will depend not only on the choice of parameters (i.e. for which lepton does $\Delta m \geq 2m_l$ hold) but also the choice of neutrino model. We will consider three distinct possible final states: (i) e^+e^- , i.e. $\Delta m \geq 2m_e$ for Dirac neutrinos, (ii) $\nu\bar{\nu}$, i.e. $\Delta m < 2m_e$ for Dirac neutrinos, and (iii) $\nu\bar{\nu}$ for Majorana neutrinos.

(i) If we consider the SM neutrino to be a Dirac particle, then the upper bound on light neutrino masses implies a Yukawa coupling of $y_\nu \lesssim 10^{-11}$ [56]. Thus when decays into charged leptons are kinematically allowed ($\Delta m \geq 2m_e$), their relatively large coupling in the Yukawa sector will render decays into neutrinos subdominant. There is the important constraint that $\Delta m < 2m_\mu$, as should $\mu^+\mu^-$ pairs be produced, their Yukawa is large enough that for no allowed values of Δm and $\langle\phi\rangle$ would $\tau_{\chi^*} > 0.1$ Gyr. Thus for $\Delta m \geq 2m_e$, decays to e^+e^- will dominate. A representative region of parameter space can be seen in Fig. 7. To obtain the correct relic abundance, parameters must lie on the dashed line. We find that for $m_\chi \sim 600$ GeV, the breaking scale $\langle\phi\rangle$ is required to be in the ~ 10 MeV range, while (for $\epsilon = 10^{-5}$) Δm is in the MeV. It should be noted that these parameters coincide with an $x_{sc-f} \sim 1$, which is well above $T_f^{\chi-\chi^*} \sim \Delta m$, removing the possibility of depopulation of the heavier DM state.

Interestingly, the (1-30) Gyr lifetime range preferred by Abdelqader and Melia [35] has been nearly completely excluded for decays into charged particles, leaving only the restrictive region of (0.1-1) Gyr available. It should be noted however that should decays to neutrinos dominate, we can avoid this exclusion region entirely.

(ii) Should $\Delta m < 2m_e$ only neutrinos are kinematically available. As Dirac neutrinos couple only very weakly with the SM Higgs, the lifetime of the DM will be too long to affect structure formation. There are two ways in which we could reduce τ_{χ^*} , by either increasing α or decreasing m_ϕ . We find, however, that for $m_\phi > \Delta m$, there are no values of α and m_ϕ that can yield a lifetime short enough. If, however, $m_\phi \lesssim \Delta m$, the process $\chi^* \rightarrow \chi + \phi'$ becomes kinematically allowed. The rate for this process does not contain the high level of suppression that decays into SM final states suffer, and we find its lifetime to be $\ll 0.1$ Gyr, dominating over decays into $\nu\bar{\nu}$. Thus for the choice of parameters $m_\phi > \Delta m$ the DM lifetime is too long, and for $m_\phi \lesssim \Delta m$ $\nu\bar{\nu}$ fi-

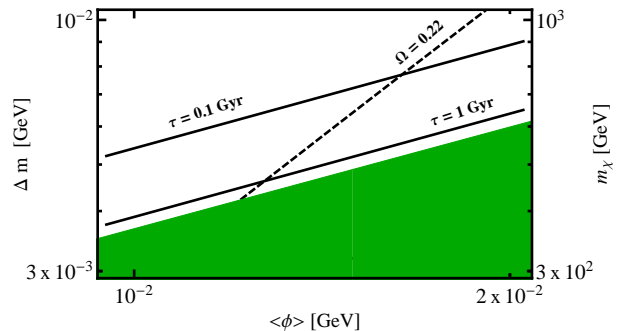


FIG. 7. Available parameter space for decays into e^+e^- ($y_l = y_e$) for $\lambda_\phi = 1$ and $\alpha = 10^{-5}$ for lifetimes $\tau_{\chi^*} = 0.1$ Gyr (solid black upper line), and $\tau_{\chi^*} = 1$ Gyr (solid black lower line). Parameters yielding correct freeze-out abundance lie on the dashed black line. Shaded is the exclusion region from [43]. We have chosen $\epsilon = 10^{-5}$.

nal states are unimportant, and τ_{χ^*} is far too short. It therefore seems that in no region of parameter space can decays into Dirac neutrinos affect structure formation.

(iii) Should we introduce Majorana masses for the ν_R and employ the type I see-saw mechanism, we have the freedom to make y_ν large enough (while still keeping the neutrino mass small) such that decays with neutrino final states will dominate without the need for fine-tuning m_ϕ . We can consider three options: $y_\nu^2 \ll y_e^2$, $y_\nu^2 \simeq y_e^2$, and $y_\nu^2 \gg y_e^2$. Should $y_\nu^2 \simeq y_e^2$ or $y_\nu^2 \ll y_e^2$, decays into electrons are either important or dominate, and so allowed parameters will be the same as for the Dirac case. However, for $y_\nu^2 \gg y_e^2$, neutrino final states are dominant for all values of Δm , and while we still need to respect the observational constraints in Fig. 7, we have a wider parameter space available, an example of which can be seen in Fig. 8.

Conversely to before, having $\Delta m \geq 2m_e$ makes $\tau_{\chi^*} > 0.1$ Gyr impossible, as the Yukawa controlling the decay is much larger than that of the electron. This constraint requires us to choose much smaller values of m_ϕ than for e^+e^- final states, if we wish to maintain thermal production. The smaller the value of m_ϕ , the larger the $\chi-\chi$ cross section in present-day halos. In [55] authors argued that to maintain the observed ellipticity in galactic halos, the timescale for DM self-interactions must be longer than the halo age ($\Gamma_{DM-DM}^{-1} > 10^{10}$ yr). Following the approach in [55] constraints were placed on our parameter space, resulting in the shaded exclusion region in Fig. 8.

IV. CONCLUSIONS

Models for decaying dark matter are interesting in that they maintain the attractive features of the Λ CDM model, while alleviating the issues pertaining to the over

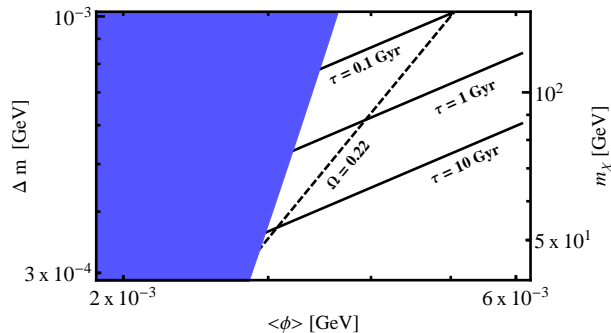


FIG. 8. Available parameter space for decays into $\bar{\nu}\nu$ with a larger Yukawa of $y_\nu \simeq 10^{-4}$ (ν are Majorana) for $\lambda_\phi = 0.8$ and $\alpha = 10^{-5}$ for lifetimes $\tau_{\chi^*} = 0.1$ Gyr (solid black upper line), $\tau_{\chi^*} = 1$ Gyr (solid black center line) and $\tau_{\chi^*} = 10$ Gyr (solid black lower line). Parameters yielding correct freeze-out abundance lie on the dashed black line. Shaded is the exclusion region based on ellipticity constraints [55]. We have chosen $\epsilon = 0.7 \times 10^{-5}$.

prediction of small scale power. In this work we investigated two examples of the class of DM models in which decay occurs via the process $\chi^* \rightarrow \chi + l$, where χ^* and χ are nearly degenerate in mass (in this work we chose $\Delta m/m_{\chi^*} \equiv \epsilon \simeq 10^{-5}$) and l is relativistic.

In the first scenario, we considered the possibility of decays into SM final states. We demonstrated that through the breaking of a discrete \mathbb{Z}_4 symmetry with the real scalar field ϕ , we could both produce two Majorana DM candidates χ^* and χ with nondegenerate mass, and allow for the decay channel $\chi^* \rightarrow \chi + \text{SM}$. The required long lifetime [(0.1-100) Gyr] was naturally achieved, as the decay rate was suppressed by a high power of ϵ , by small Yukawa couplings, and by the small mixing between SM-sector and dark-sector particles.

The only two viable decay modes involving SM final states were $\chi^* \rightarrow \chi + e^+e^-$ and $\chi^* \rightarrow \chi + \bar{\nu}\nu$, where the latter is possible only in the case of Majorana neutrinos.

We found that for DM masses in the range (50-800) GeV [$\Delta m \simeq (0.4 - 8)$ MeV] and for a ϕ -Higgs mixing of $\alpha \simeq 10^{-5}$, all required criteria, including thermal production, could be met if the \mathbb{Z}_4 symmetry was broken at the MeV scale, with $\langle\phi\rangle \simeq (3 - 20)$ MeV ($m_\phi \simeq (2 - 20)$ MeV). Interestingly, in applying the constraints on decays to e^+e^- derived in [43], we showed that this final state is almost excluded for DM lifetimes in the (1-30) Gyr range preferred in [35]. Dirac neutrinos were unable to fulfill the requirements for decaying DM, as their Yukawa coupling is too small. Thus decays to Majorana neutrinos are preferred by such DM decay models, as they are not constrained to either the short lifetimes applicable for e^+e^- decays nor the small Yukawa couplings of Dirac neutrinos.

In the second scenario, we considered the possibility of non-SM final states. This was achieved by replacing the discrete \mathbb{Z}_4 symmetry with a continuous $U(1)$ symmetry. Breaking of the $U(1)$ symmetry led to a pseudo-Nambu-Goldstone boson, which became the light final state produced in decays. As the DM decay process was no longer strongly suppressed, we were forced to finely tune model parameters to obtain a DM lifetime in the correct range. A consequence of this fine-tuning was to make DM production via mixing with the SM no longer possible. In this scenario, the dark and visible sectors are almost decoupled from each other. Though aesthetically less appealing, this model demonstrates the feasibility of decaying dark matter, independent of the strength of coupling to the SM.

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